DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 223.3 MIDTERM EXAM November 1, 2004, 16:30 to 18:00

The marks for each question are shown as [X]. Closed Book. No electronic devices.

The exam is in three parts. Use different Exam Booklets for each separate part and label the booklets PART A, PART B, and PART C. Please show all of your work.

PART A

Answer the following questions in an Exam Booklet and label it PART A.

1. [5] Find an equation of the plane containing the line $x + 2y + 4z = 21$, $x - y + 6z = 13$ and the line $x = 2 + 3t$, $y = 4$, $z = -3 + 5t$.

2. [5] In what direction is the rate of change of $f(x, y, z) = xyz$ smallest at the point $(2, -1, 3)$? What is that rate of change?

3. [5] Find the rate of change in $f(x, y, z) = x^2y + xy^3z$ at $(2, -1, 2)$ with respect to distance traveled along the curve $x^2 - y^2 = 3$, $z = x$ in the direction of increasing x

PART B

Answer the following questions in an Exam Booklet and label it PART B.

4. [5] Calculate the normal component a_N of the acceleration of a particle if its position is given by $x = t^2 + 1$, $y = 2t^2 - 1$, $z = t^2 + 5t$, $t \geq 0$ (t being time).

5. [5] If $z = x^2 + xy + y^2 \sin(x/y)$, find all of the first and second partial derivatives.

6. [5] Find $\partial z/\partial t)_s$ if $z = f(x, y)$, $x = g(r)$, $y = h(r)$, $r = k(s, t)$ where $z = e^{x+y}$, $x = 2r + 5$, $y = 2r - 5$, $r = t \ln(s^2 + t^2)$.

PART C

Answer the following questions in an Exam Booklet and label it PART C.

7. [5] Calculate the unit vectors $\hat{\mathbf{T}}$, $\hat{\mathbf{N}}$, and $\hat{\mathbf{B}}$ for the curve $x = \cos t$, $y = \sin t$, $z = t$.

8. [10] (a) Find all critical points of $f(x, y) = x^3 + y^3 - 3x - 3y + 2$ in the disc $x^2 + y^2 <$

(b) Use the parametric equations $x = \cos(\theta)$, $y = \sin(\theta)$ to find maximum and minimum values of f on $x^2 + y^2 = 1$.